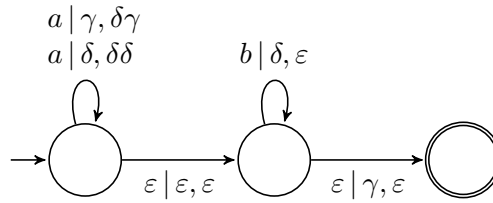


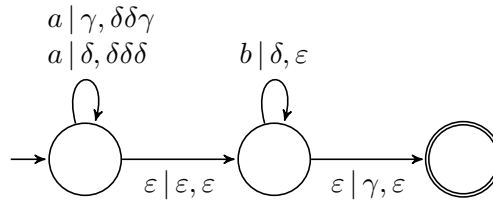
ΑΣΚΗΣΕΙΣ ΓΙΑ ΑΥΤΟΜΑΤΑ ΣΤΟΙΒΑΣ

Βρείτε τα αυτόματα στοίβας για τις παρακάτω γλώσσες:

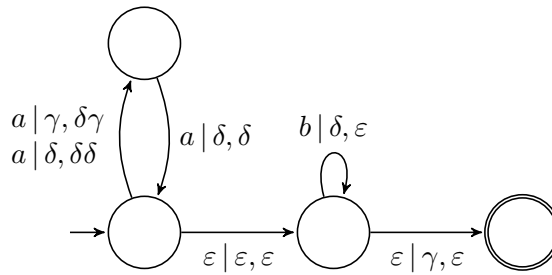
(1) $L = \{a^i b^i : i \geq 0\}$.



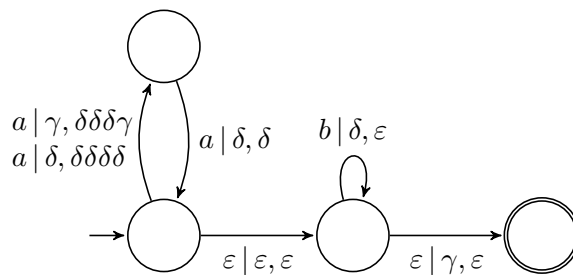
(2) $L = \{a^i b^{2i} : i \geq 0\}$.



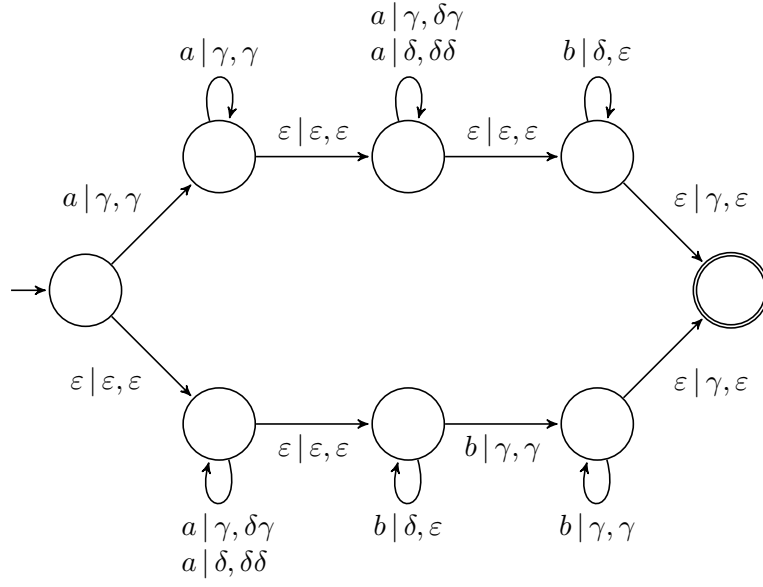
(3) $L = \{a^{2i} b^i : i \geq 0\}$.



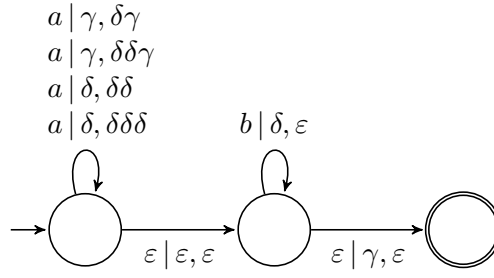
(4) $L = \{a^{2i} b^{3i} : i \geq 1\}$.



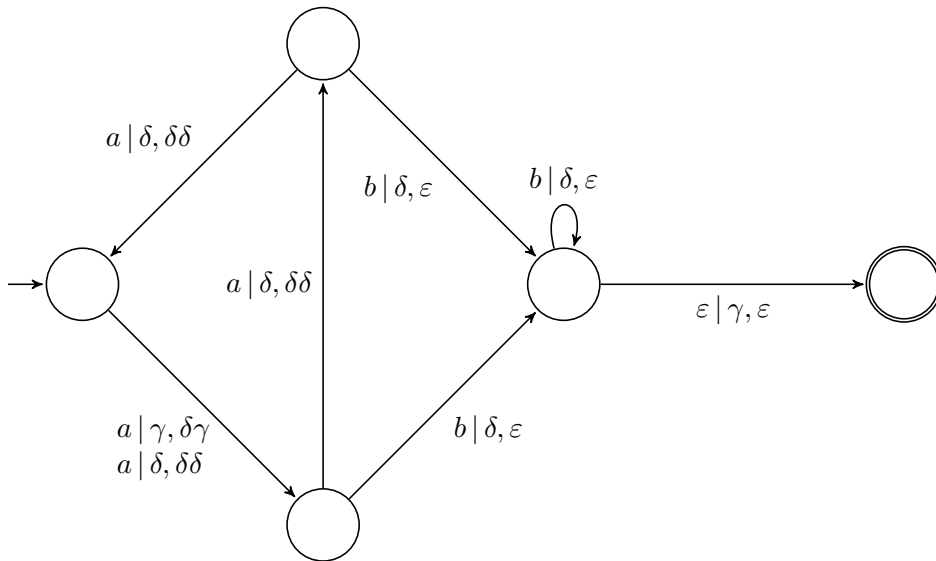
(5) $L = \{a^i b^j : i \neq j, i, j \geq 0\}$.



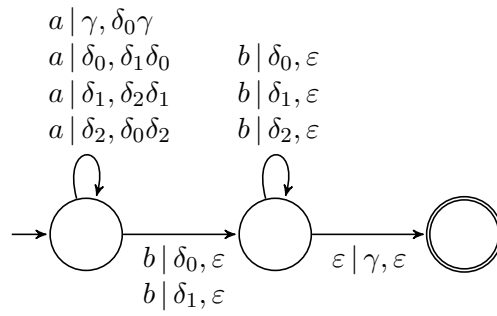
(6) $L = \{a^i b^j : 0 \leq i \leq j \leq 2i\}$.



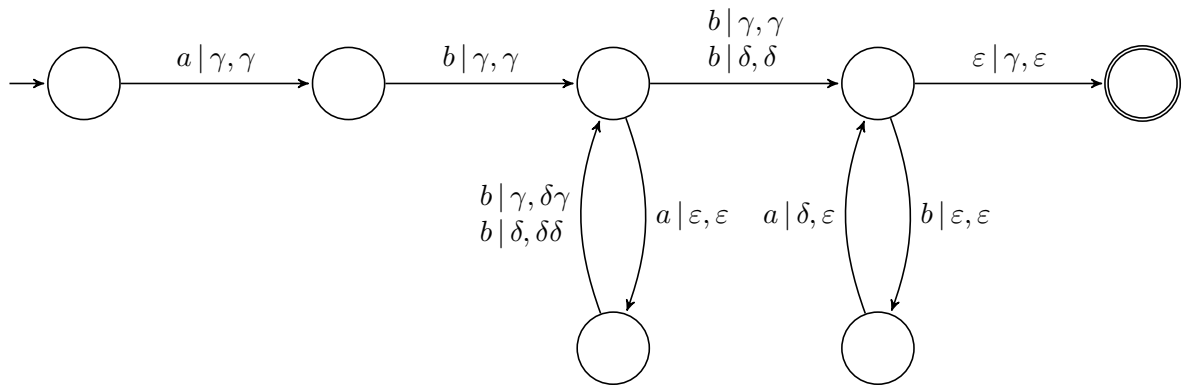
(7) $L = \{a^i b^i : i \neq 0 \pmod{3}\}$.



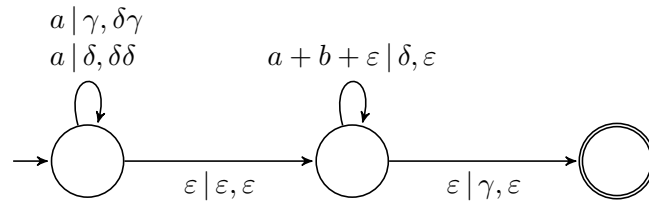
Εναλλακτική λύση:



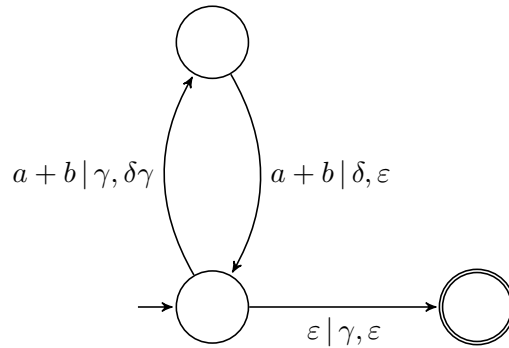
(8) $L = \{ab(ab)^i b(ba)^i : i \geq 0\}$.



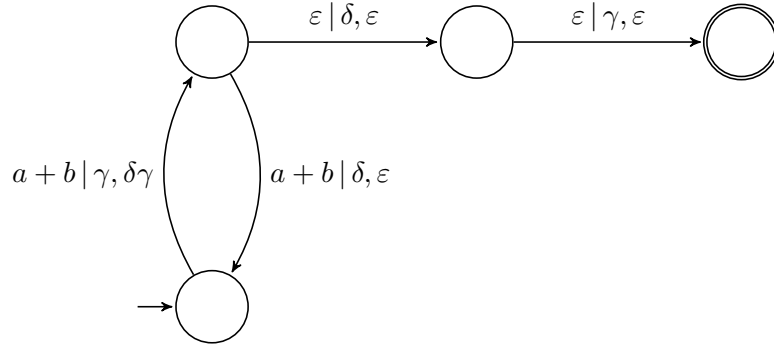
(9) $L = \{a^n w : n \geq 0, w \in (a+b)^* \text{ και } |w| \leq n\}$.



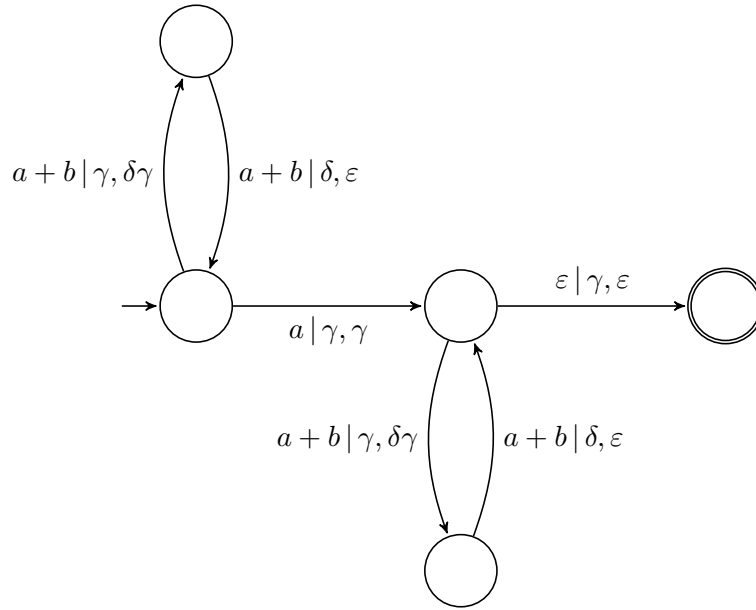
(10) $L = \{w \in (a+b)^* : |w| \text{ άρτιο}\}$.



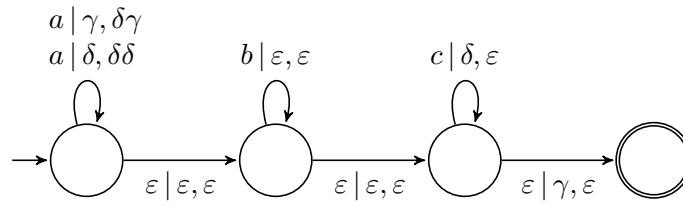
(11) $L = \{w \in (a+b)^*: |w| \text{ περιττό}\}.$



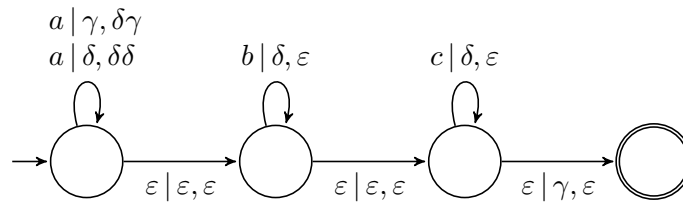
(12) $L = \{w_1aw_2 \in (a+b)^*: |w_1| \text{ και } |w_2| \text{ άρτια}\}.$



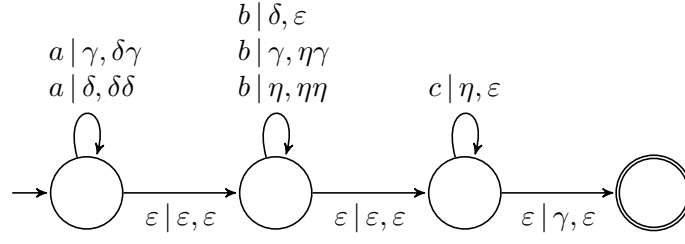
(13) $L = \{a^ib^jc^i: i, j \geq 0\}.$



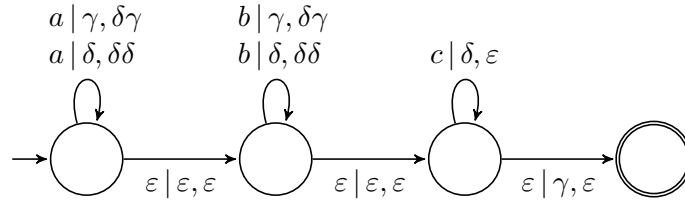
(14) $L = \{a^ib^jc^k: i = j+k\}.$



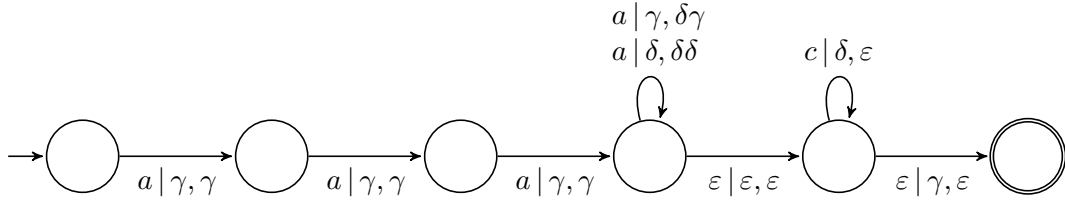
(15) $L = \{a^i b^j c^k : j = i + k\}.$



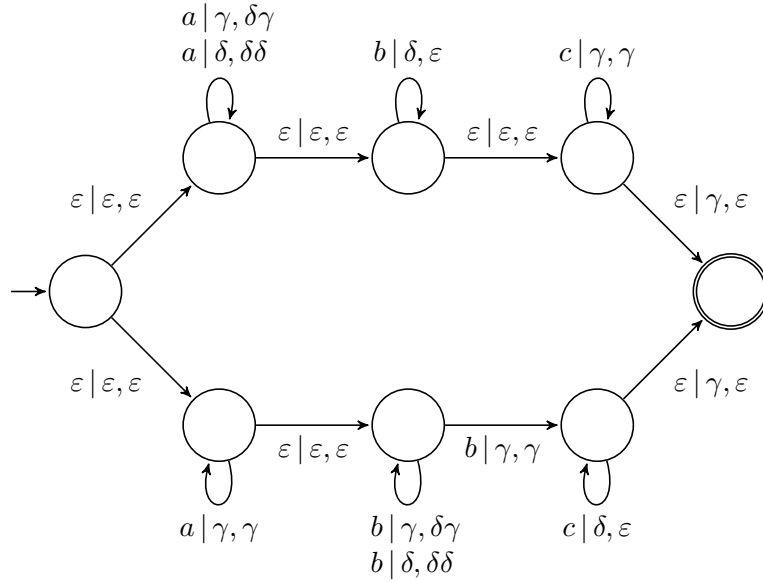
(16) $L = \{a^i b^j c^k : k = i + j\}.$



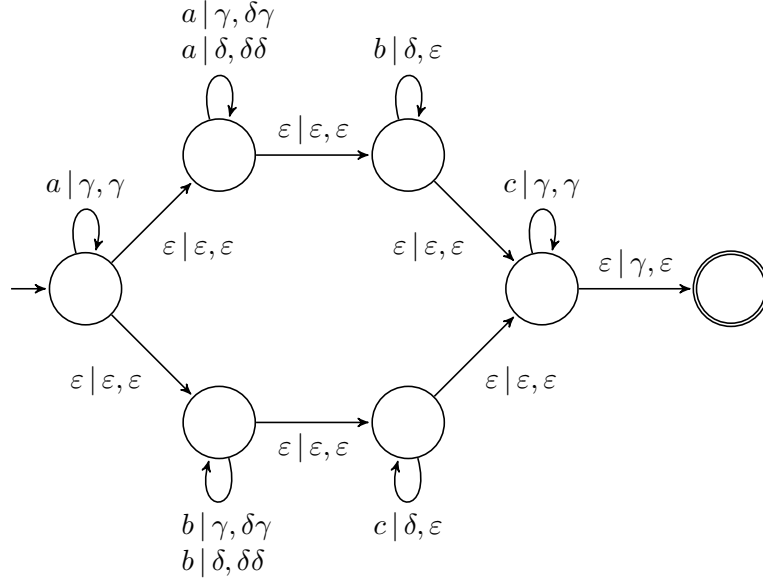
(17) $L = \{a^3 b^i c^i : i \geq 0\}.$



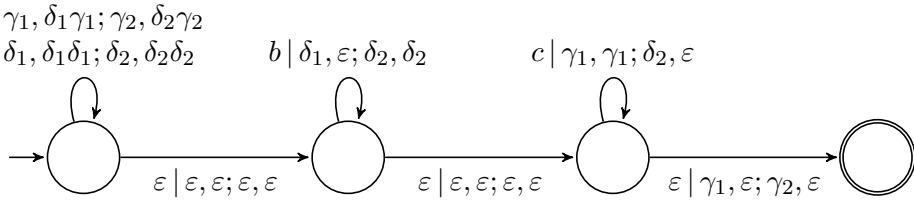
(18) $L = \{a^i b^j c^k : i = j \text{ } \dot{\vee} \text{ } j = k\}.$



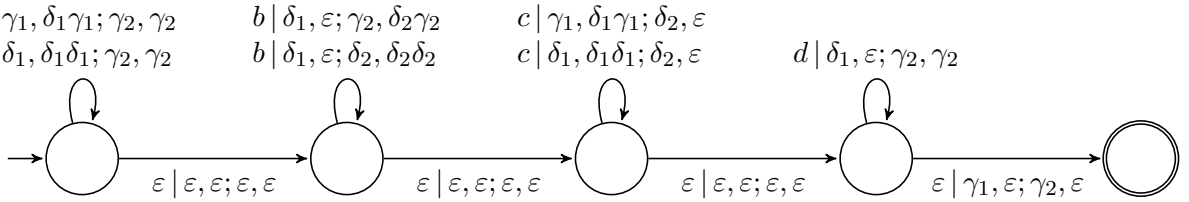
(19) $L = \{a^i b^j c^k : j < i \text{ and } j < k\}.$



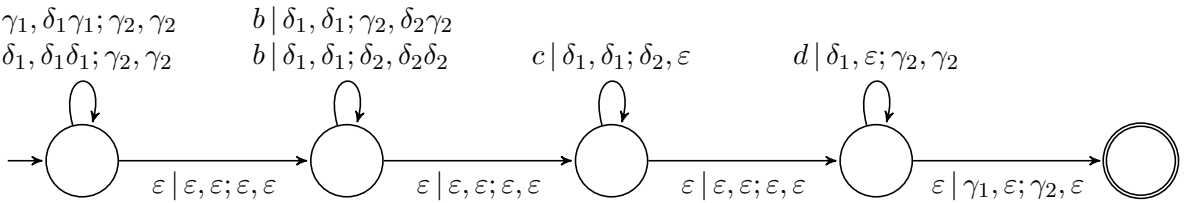
(20) $L = \{a^i b^i c^i : i \geq 0\}.$



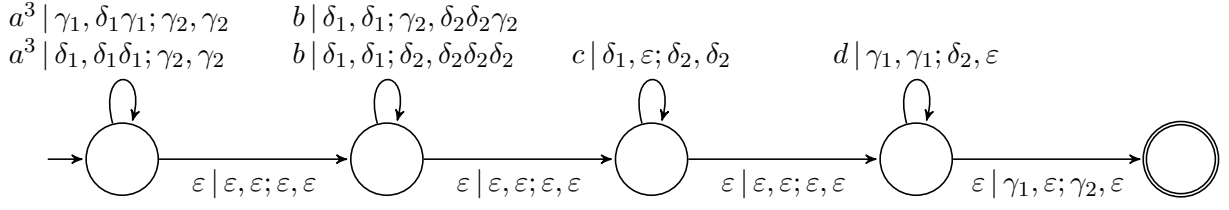
(21) $L = \{a^i b^i c^i d^i : i \geq 0\}.$



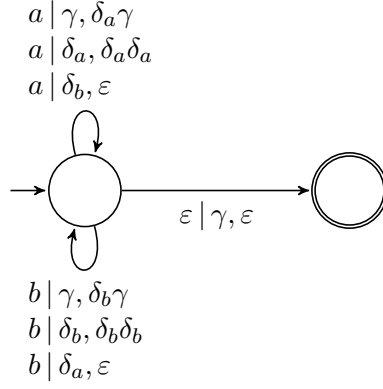
(22) $L = \{a^i b^j c^j d^i : i, j \geq 0\}.$



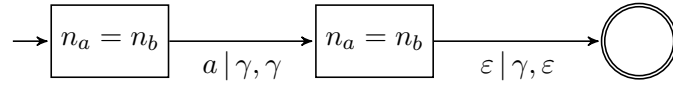
(23) $L = \{a^{3i}b^j c^i d^{2j}: i, j \geq 0\}$.



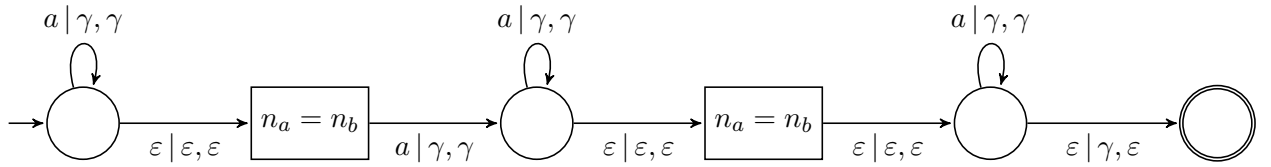
(24) $L = \{w \in (a+b)^*: n_a(w) = n_b(w)\}$.



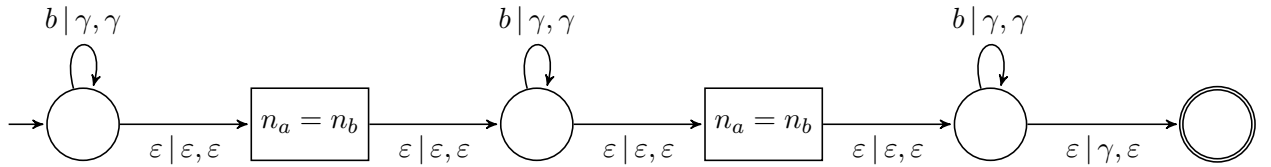
(25) $L = \{w \in (a+b)^*: n_a(w) = n_b(w) + 1\}$.



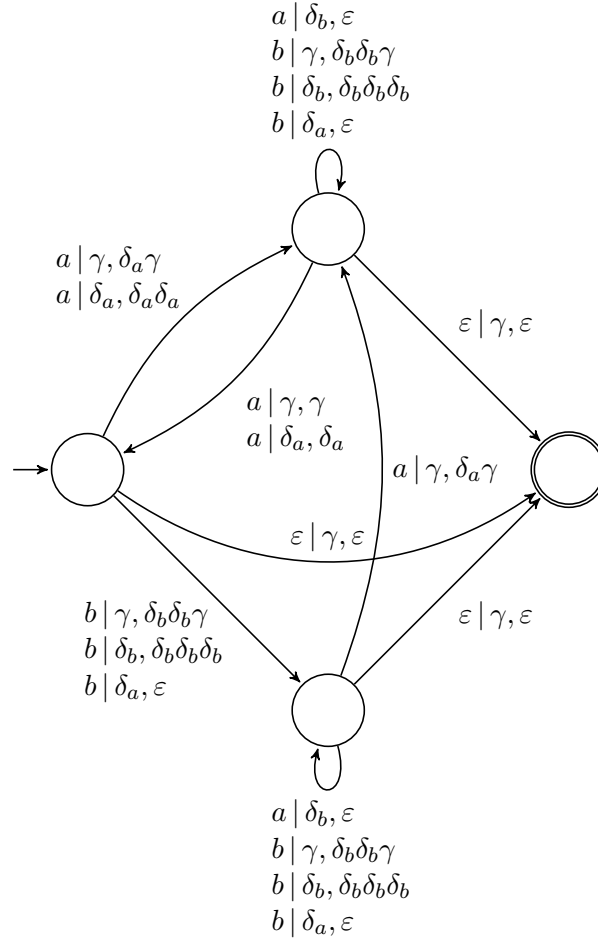
(26) $L = \{w \in (a+b)^*: n_a(w) \geq n_b(w) + 1\}$.



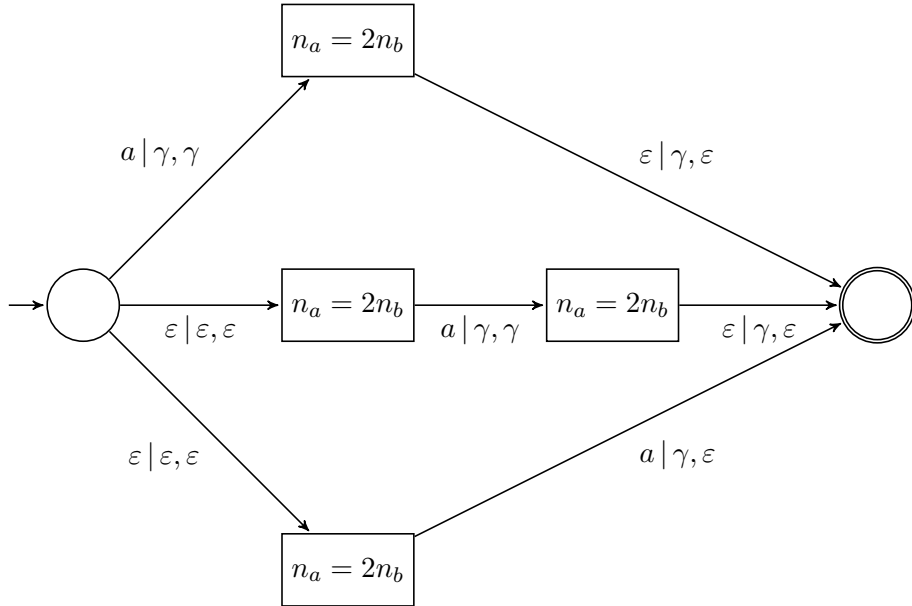
(27) $L = \{w \in (a+b)^*: n_a(w) < n_b(w)\}$.



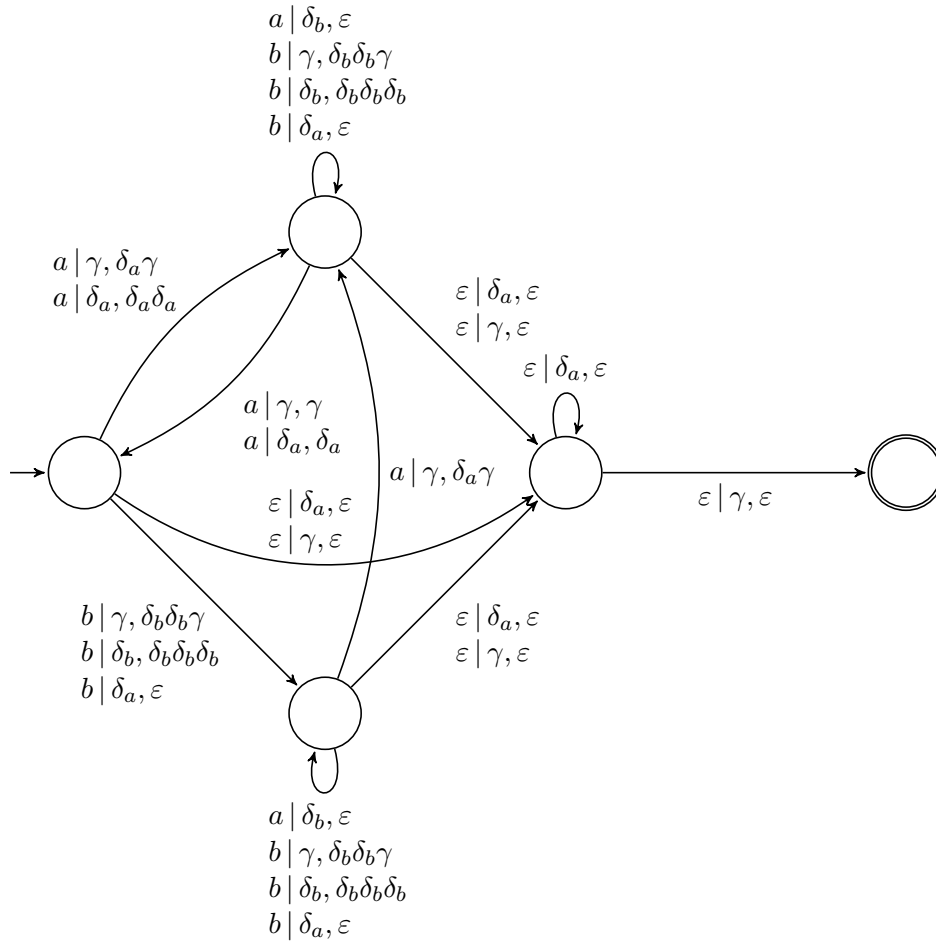
(28) $L = \{w \in (a+b)^*: n_a(w) = 2n_b(w)\}.$



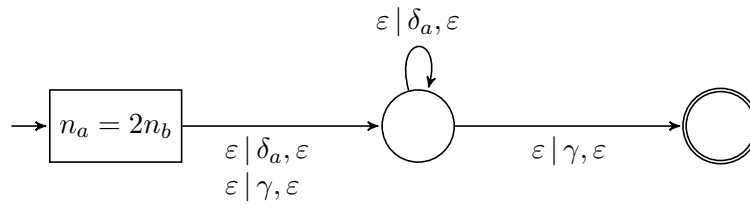
(29) $L = \{w \in (a+b)^*: n_a(w) = 2n_b(w) + 1\}.$



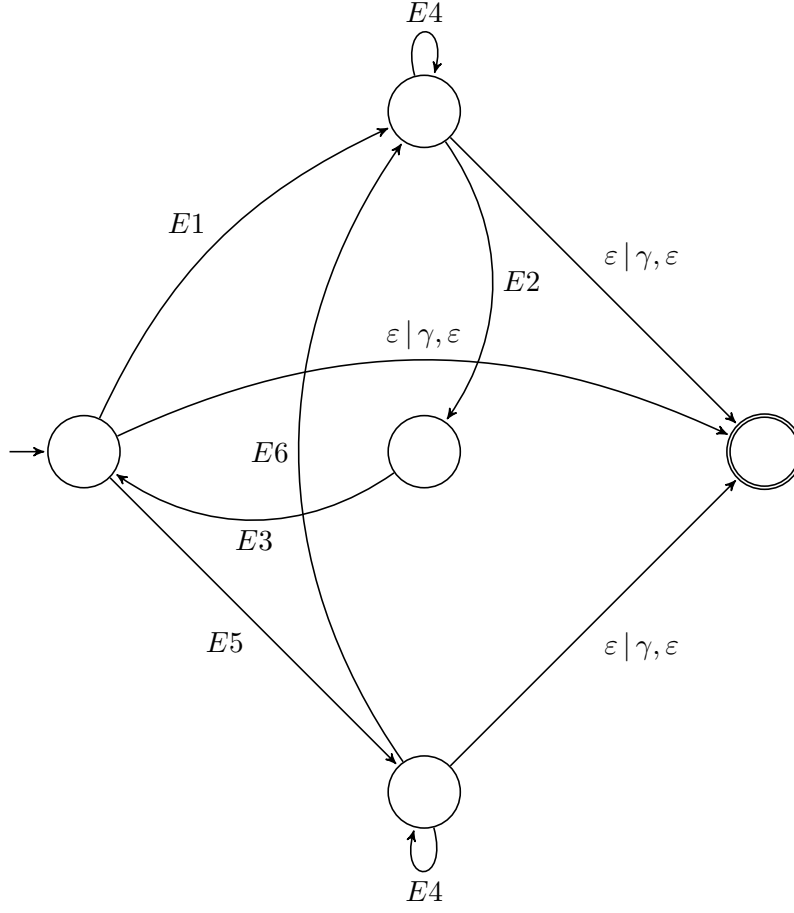
(30) $L = \{w \in (a+b)^*: n_a(w) > 2n_b(w)\}.$



Η (συμβολικά):



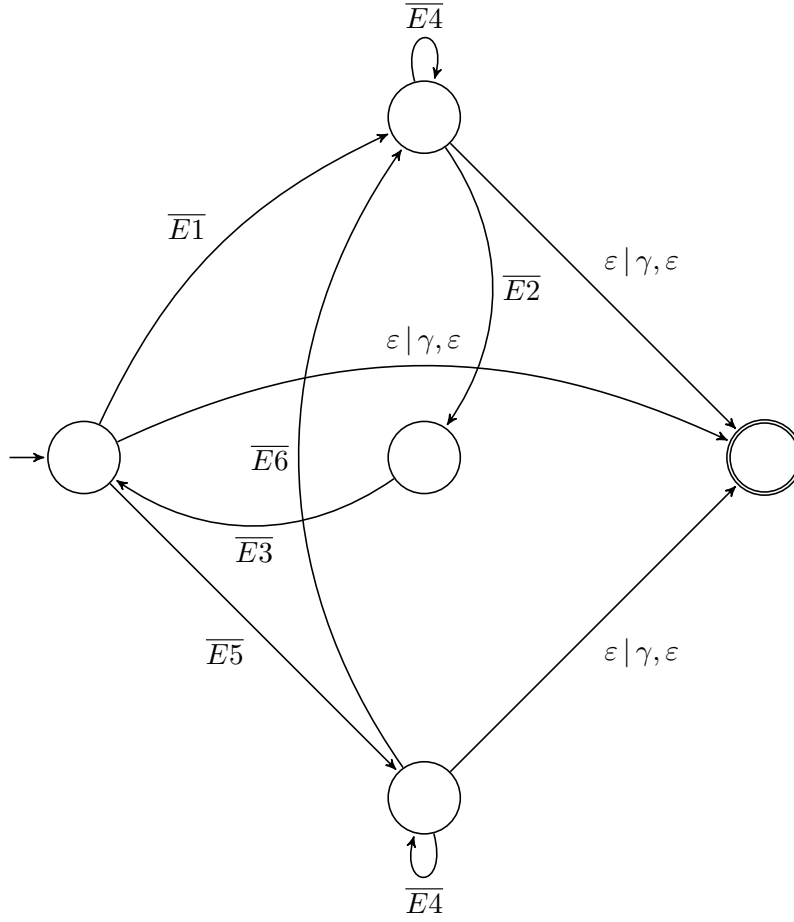
$$(31) \ L = \{w \in (a+b)^*: n_a(w) = 3n_b(w)\}.$$



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$E1 : a \mid \gamma, \delta_a \gamma$	$E4 : b \mid \gamma, \delta_b \delta_b \delta_b \gamma$
$a \mid \delta_a, \delta_a \delta_a$	$b \mid \delta_b, \delta_b \delta_b \delta_b \delta_b$
$E2 : a \mid \gamma, \gamma$	$b \mid \delta_a, \varepsilon$
$a \mid \delta_a, \delta_a$	$a \mid \delta_b, \varepsilon$
$E3 : a \mid \gamma, \gamma$	$E5 : b \mid \gamma, \delta_b \delta_b \delta_b \gamma$
$a \mid \delta_a, \delta_a$	$b \mid \delta_b, \delta_b \delta_b \delta_b \delta_b$
$\varepsilon \mid \varepsilon, \varepsilon$	$b \mid \delta_a, \varepsilon$
	$E6 : a \mid \gamma, \delta_a \gamma$

$$(32) \ L = \{w \in (a+b)^*: 2n_a(w) \leq n_b(w) \leq 3n_a(w)\}.$$



όπου

$$\begin{aligned} \overline{E1} : & b \mid \gamma, \delta_b \gamma \\ & b \mid \delta_b, \delta_b \delta_b \end{aligned}$$

$$\begin{aligned} \overline{E2} : & b \mid \gamma, \gamma \\ & b \mid \delta_b, \delta_b \end{aligned}$$

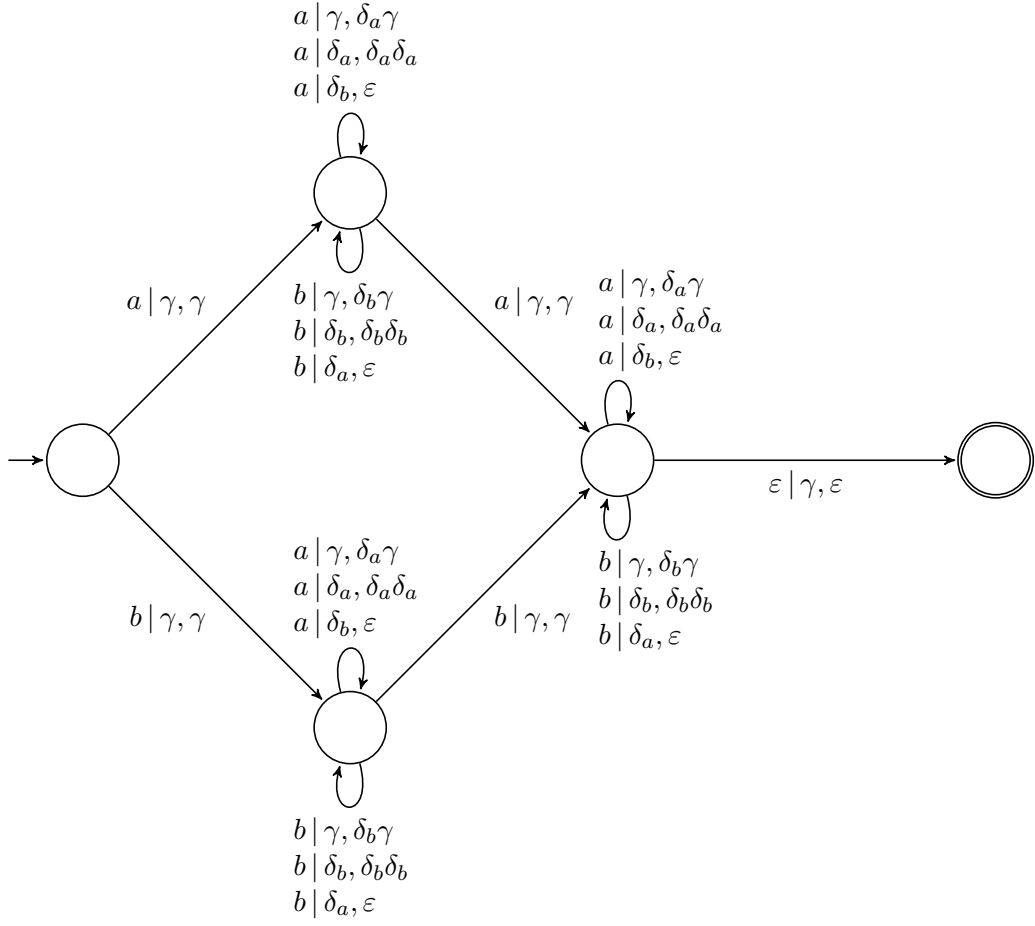
$$\begin{aligned} \overline{E3} : & b \mid \gamma, \gamma \\ & b \mid \delta_b, \delta_b \\ & \varepsilon \mid \varepsilon, \varepsilon \end{aligned}$$

$$\overline{E6} : b \mid \gamma, \delta_b \gamma$$

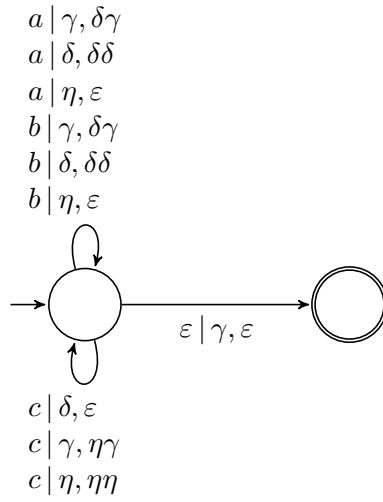
$$\begin{aligned} \overline{E4} : & b \mid \gamma, \delta_b \delta_b \delta_b \gamma \\ & a \mid \gamma, \delta_a \delta_a \delta_a \delta_a \gamma \\ & b \mid \delta_b, \delta_b \delta_b \delta_b \delta_b \\ & a \mid \delta_a, \delta_a \delta_a \delta_a \delta_a \delta_a \\ & a \mid \delta_b, \varepsilon \\ & b \mid \delta_a, \varepsilon \end{aligned}$$

$$\begin{aligned} \overline{E5} : & a \mid \gamma, \delta_a \delta_a \delta_a \gamma \\ & a \mid \gamma, \delta_a \delta_a \delta_a \delta_a \gamma \\ & a \mid \delta_a, \delta_a \delta_a \delta_a \delta_a \\ & a \mid \delta_a, \delta_a \delta_a \delta_a \delta_a \delta_a \end{aligned}$$

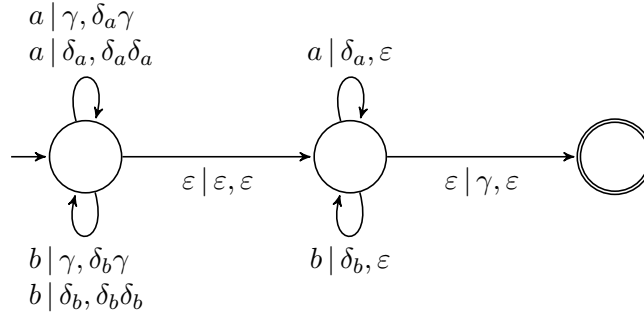
(33) $L = \{w \in (a+b)^*: |n_a(w) - n_b(w)| = 2\}.$



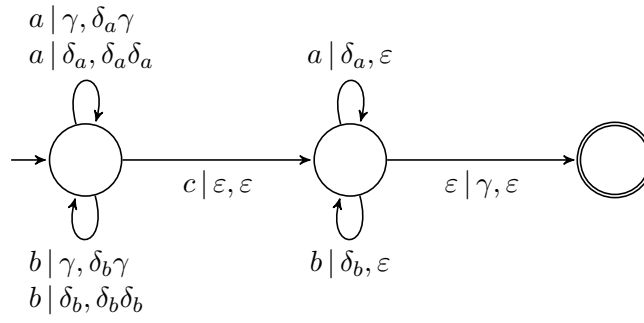
(34) $L = \{w \in (a+b+c)^*: n_a(w) + n_b(w) = n_c(w)\}.$



(35) $L = \{ww^R: w \in (a+b)^*\}$.



(36) $L = \{wcw^R: w \in (a+b)^*\}$.



(37) $L = \{w \in (a+b)^*: w = w^R\}$.

Απ.: Προφανώς, ισχύει ότι $w = w^R$ αν και μόνον αν είτε $w = zz^R$ ή $w = zσz^R$, για κάποια λέξη $z \in (a+b)^*$ και κάποιο σύμβολο $σ = a$ ή b . Επομένως:

