ON MANIFOLDS WITH ALMOST HYPERCOMPLEX STRUCTURES AND ALMOST CONTACT 3-STRUCTURES, EQUIPPED WITH METRICS OF HERMITIAN-NORDEN TYPE

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The report provides an overview of the author's latest results on this topic.

In the beginning, some facts are given concerning the almost hypercomplex manifolds with Hermitian-Norden metrics known from the papers of the author, K. Gribachev and collaborators.

Next, integrable hypercomplex structures with Hermitian-Norden metrics on Lie groups of dimension 4 are considered. The corresponding five types of invariant hypercomplex structures with hyper- Hermitian metric are constructed here. The different cases regarding the signature of the basic pseudo-Riemannian metric are considered.

Further, the tangent bundle of an almost Norden manifold and the complete lift of the Norden metric are considered as a $4n$-dimensional manifold. It is equipped with an almost hypercomplex Hermitian-Norden structure and characterized geometrically. The case when the base manifold is an h-sphere is considered.

Then, it is introduced an associated Nijenhuis tensor of endomorphisms in the tangent bundle of an almost hypercomplex manifold with Hermitian-Norden metrics. There are studied relations between the six associated Nijenhuis tensors of an almost hypercomplex structure as well as their vanishing. It is given a geometric interpretation of the associated Nijenhuis tensors for an almost hypercomplex structure and Hermitian- Norden metrics. Finally, an example of a 4-dimensional manifold of the considered type with vanishing associated Nijenhuis tensors is given.

Next, quaternionic Kähler manifolds corresponding to almost hypercomplex manifolds with Hermitian-Norden metrics are considered. Some necessary and sufficient conditions for the studied manifolds to be isotropic hyper-Kählerian and flat are found. It is proved that the quaternionic Kähler manifolds with the considered metric structure are Einstein for dimension at least 8. The class of the non-hyper-Kähler quaternionic Kähler manifold of the considered type is determined.

In the second part of this report, it is introduced a differentiable manifold with almost contact 3-structure which consists of an almost contact metric structure and two almost contact B-metric structures. The corresponding classifications are discussed. The product of this manifold and a real line is an almost hypercomplex manifold with Hermitian-Norden metrics. It is proved that the introduced manifold of cosymplectic type is flat. Some examples of the studied manifolds are given.

Finally, it is considered a differentiable manifold equipped with a pseudo-Riemannian metric and an almost contact 3-structure so that one almost contact metric structure and two almost contact B-metric structures are generated. There are introduced associated Nijenhuis tensors for the studied structures. The vanishing of the Nijenhuis tensors and their associated tensors is considered. It is given a geometric interpretation of the vanishing of associated Nijenhuis tensors for the studied structures as a necessary and sufficient condition for existence of affine connections with totally skew-symmetric torsions preserving the structure. An example of a 7-dimensional manifold with connections of the considered type is given.